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TECHNICAL REPORT

A SONIC METHOD OF MEASURING  
YOUNG'S MODULUS OF ELASTICITY AT HIGH TEMPERATURE

BY

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### ABSTRACT

A method of measuring Young's modulus of elasticity employing a sonic technique is described. This method has the advantage of being usable over a fairly wide range of temperatures, is non-destructive, and is capable of reproducing values to within  $\pm 1\%$ . An audio frequency generator is used to drive a long thin bar suspended by a platinum wire from a speaker cone. Another wire attached to a variable reluctance pick-up needle suspends the bar at the other end. An oscilloscope is used to detect the resonant frequency of the bar from the pick-up device. A simple relationship reported by Pickett<sup>4</sup> is used to relate the resonant frequency of the bar to Young's modulus.

## INTRODUCTION

The elastic properties of solids must be known in many problems encountered in the designing and engineering of mechanical systems, particularly those involving vibration and stress analysis. This laboratory is concerned for example, with the problem of thermal shock. In order to know the conditions of size, shape and temperature which must prevail for a brittle solid to fracture due to thermal stresses, it is necessary to know the Young's modulus\* of the material as a function of temperature. For some quite elastic substances it is possible to measure strain directly by pulling on either end of a bar and noting its elongation. But many materials, particularly ceramic materials, fail before they show any detectable change in length. What should a designer do when he must measure Young's modulus of such brittle materials? Even if, by very delicate measurements, strain can be detected in a specimen by conventional methods, it may have been strained beyond its elastic limit. This is the situation with many so-called "destructive" methods, which also have the added disadvantage of poor precision. Fortunately there exists a method for determining Young's modulus using extremely small displacements which are well within the elastic limit. This method relates Young's modulus to the resonant properties of the tested material.<sup>1,2</sup>

\* Young's modulus is the ratio of a linear stress to the strain it produces.

<sup>1</sup> Baab, K.E., and Kraner, H.M., "Sonic Method of Determining Young's Modulus of Elasticity", J. Am. Ceram. Soc. Vol. 31, 1948. p. 318.

<sup>2</sup> Govt. Contract W33-038 ac 16374, Penn. State College.

#### DISCUSSION OF METHOD

It is well known<sup>3</sup> that the frequency of a vibrating bar depends on its length, cross section, density, and Young's modulus. The bar should be supported to vibrate freely when excited with a periodic driving impulse, and the resulting resonance detected. The resonant frequency of the bar is related to the modulus of elasticity by a simple relationship, once the proper correction factors have been determined.<sup>4</sup> In order that the supports interfere as little as possible with the free motion of the bar, it should be supported at the nodes, i.e., the points of least motion. Such supports are usually rigid knife edges; the bar rests on these, is tapped at one end by the driver, and excites a phonograph type pick-up at the other end. If a wire suspension is used it has the added advantage that the suspending wires may act as driver and pick-up. To avoid interference by vibration of the wire, frequencies which are above the strong harmonics of the wire are used or transverse damping provided for the wire. For purposes of measurement at high temperature, 30 gage platinum wire is satisfactory for supporting an alumina bar through the top of a small electric furnace.

The advantages of such a system are obvious: Standard electronic gear (See Fig. 1) is all that is required for measurement, the specimen is not damaged, and the precision is good. The chief disadvantage is that the operator must have long slender bars of the material, made without unduly affecting its elastic properties, i.e., without changing its density in any region.

<sup>3</sup>Morse, P.M., "Vibration & Sound", 2d ed. N.Y. McGraw - Hill Book Co., 1948, and Rayleigh, baron, John William, Strutt, "Theory of Sound" Vol. I & II, 2d ed. rev. & enl. New York Dover Publisher, 1945.

<sup>4</sup>ASTM Proceedings Vol. 45, 1945, Pickett, Gerald, "Equations for Computing Elastic Constants From Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders", Pg. 846-865.

EXPERIMENTAL PROCEDURE

The high alumina bars used by this laboratory were prepared by grinding to shape hydrostatically pressed and sintered bodies prepared by Coors Procelain Company (type AB-2 ceramic). The specimens were cylinders 1 cm. in diameter and  $12\frac{1}{2}$  cm. long with a resonant frequency of about 4 kilocycles in the first flexural mode. Ease of resonance decreases rapidly as length to depth ratio decreases. It is recommended that dimensions be chosen so the length to depth ratio is 10 or more; and that the frequency be of the order of Kilocycles - above the sympathetic frequencies of the wires and below the inaudible highs.

The experimental setup described below was developed for use on the Office of Naval Research Project. The apparatus is quite simple yet precise enough for the values required. Figure 1 is a schematic drawing of the entire setup. The speaker and pick-up were mounted on a rigid frame supported on resilient padding, and in such a way that they were adjustable parallel to the bar. The driving wire was tightly secured to the voice coil of the speaker in the manner shown in Figure 2. (Removal of the paper cone will prove less uncomfortable to the ear.) Theoretically, the best results are obtained when the amplitude of the driver is equal to the amplitude of the bar at the point at which it is driven. This may be found by trial, i.e., by placing the noose of the wire at various points between the node (0.22 times the length from the end) and the end and noting the amount of out-put. The proper location is the point of maximum amplitude. The noose of the pick-up wire should be attached firmly at a position between the other node and the end of the bar so as to produce the maximum amplitude. Usually this position is about  $\frac{1}{3}$  the distance between the node and the end of the bar. The pick-up device should be

assembled as shown in Figure 3. The cartridge must be placed in the position shown, so that the vibration of the bar and the wire is transmitted to the needle at an angle of 90 degrees to the cartridge case. Great care must be exercised to avoid kinks in the wire, as they dissipate much vibrational energy. If a crystal pick-up cartridge is used the preamplifier isn't necessary. However, its response drops off rapidly above 4 Kilocycles, and a variable reluctance type cartridge is necessary above that frequency. Of course, shielded cable must be used between pickup, preamplifier and amplifier, so that extraneous oscillating fields are minimized.

Resonance of the specimen is detected by noting the greatest amplitude on the oscilloscope as the frequency is varied.\* When the driver is suddenly shut off the pattern on the scope should briefly remain if the true resonance of the bar is being picked up. It is easy to confuse activity of the wires with that of the bar. A good check is to tap the bar and match its pitch with the audio-oscillator. (Light damping of the pick-up wire should not drastically affect resonance.) After the resonant frequency has been found it is measured with the null bridge frequency meter at the point where the first harmonic of the wave disappears from the output of the meter as detected on an oscilloscope. (See Fig. 4) It is desirable to have a good quality oscilloscope to make precision frequency measurements.

A furnace constructed of insulating brick and Globars, was used to heat the specimens. (See Fig. 5) The chamber was made as small as possible to facilitate temperature control and to reduce convection. The heated portions of the Globars were longer than the test bar, parallel to it, and mounted at equal intervals about the bar. Globars of equal resistances were connected in parallel to a variable power supply to provide uniform heating. A thermocouple was mounted with its bead very close to the bar.

\*An A.C. volt meter may be substituted for this purpose.

Sufficient time was allowed for the furnace to reach its equilibrium temperature at each selected power. Measurements were erratic unless the system had reached an equilibrium temperature. This condition could be detected by setting the frequency at resonance and noting the absence of any drift in amplitude.

### EXPERIMENTAL RESULTS

The equation of Timoshenko has been solved by Goens to give Young's modulus as a function of the frequency, mass, ratio of length to thickness, and Poisson's ratio for the bar. Pickett<sup>4</sup> has consolidated the solution into a simple equation which contains a correction factor for which he provides graphs. A typical experiment follows:

The following measurements are taken on the specimen before fastening it to the wire suspensions:

$m$  = mass of the cylindrical bar = 30.63 grs.

$l$  = length of cylindrical bar = 12.78 cm.

$d$  = average diameter cylindrical bar = 0.9484 cm.

The specimen is then fastened to the support wires at the proper location and the resonant frequencies found for each temperature. The following information is then compiled:

$f$  = frequency (at room temperature)  $4.065 \pm .005$

$\frac{d}{4l} = \frac{r}{l}$  = ratio of radius of gyration of cross-section to length of the bar = .01855

$\frac{l}{d}$  = ratio of length to diameter = 13.475

$T_1$  = Goen's correction factor\* = 1.03 (Poisson's ratio  $\nu$  = 1/6)

$C_1 = 4.1632 \times 10^{-3} \left( \frac{1}{d} \right)^3 \cdot T_1 \cdot \frac{l}{d} = 28.099 \text{ Sec}^2/\text{in}^3$

\* This value was taken from Pickett's graphs.



$$W = \text{Weight in lbs.} = .066939$$

$$E = C_1 W f^2 = 31.08 \times 10^6 - \text{psi}$$

Figure 6 is plot of resonant frequencies for the Coors composition from room temperature up to 820°C. Figure 7 is the corresponding plot of calculated E moduli between the same temperatures.

The chief source of experimental error is the frequency meter, which can be read to within two parts in a thousand or 0.2%. The total experimental error, considering the frequency meter, size and weight measurements, graph readings, etc. is estimated as one percent. Thus, the observed variation in E measurements shown below is slightly better than would be expected for separate specimens. The following room temperature E moduli were found for similar bars of the same shipment:

$$31.08 \times 10^6 \text{ psi}$$

$$31.73 \times " "$$

$$31.26 \times " "$$

$$31.42 \times " "$$

At about 820°C the resonant peak suddenly becomes broad and attenuated; the spread in observed values increasing to produce a "buckshot" pattern. Consequently, E modulus measurements of this material can not be made above this temperature with any assurance. However, this temperature corresponds to that at which the plasticity and Poisson's ratio both increase rapidly, indicating the upper limit of elasticity.

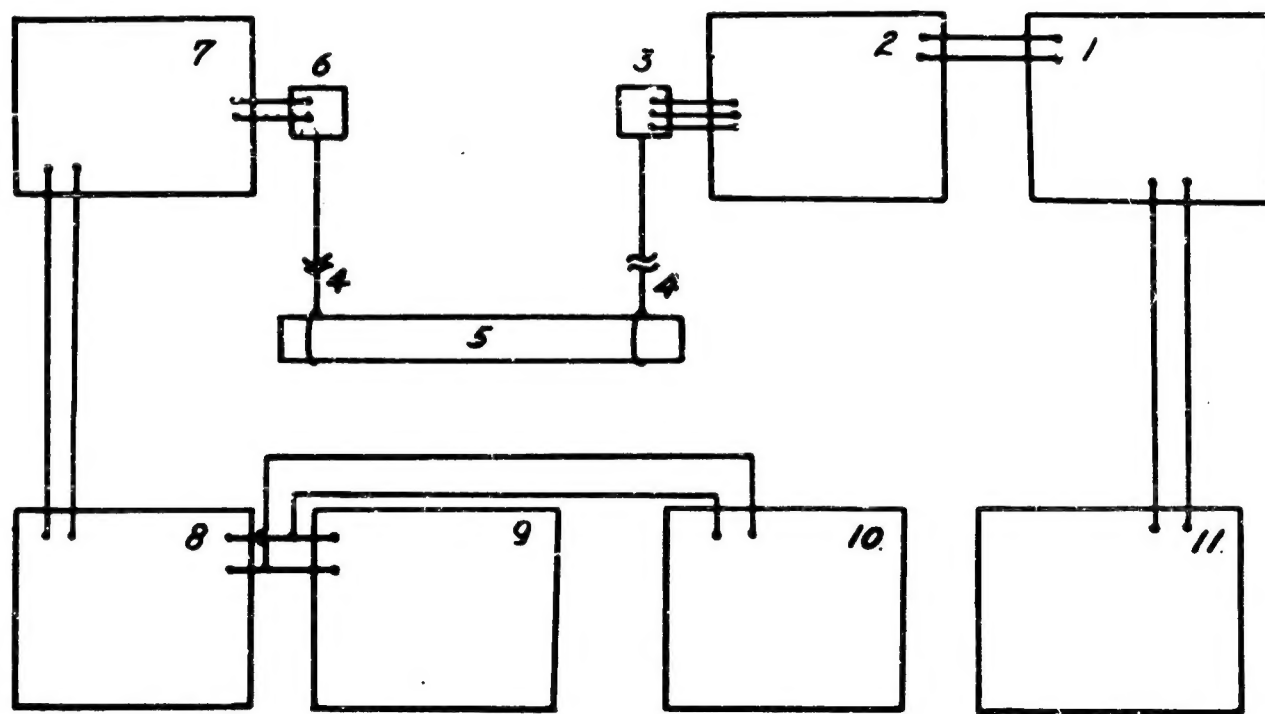


SUMMARY

A method of measuring Young's modulus of elasticity by a sonic technique has been described. The method may be used to obtain values over a fairly wide range of temperatures, is non-destructive and capable of reproducing results to within  $\pm 1\%$ . A typical experiment has been outlined and results shown for Coors type AB-2 ceramic - high strength alumina.

ACKNOWLEDGEMENT

The authors are indebted to Messers: Bassett and Ging for their aid in preparing this paper.



1. GENERAL RADIO TYPE 1302 - AUDIO OSCILLATOR WITH 600 & 5000 OHM OUTPUT IMPEDANCES TO 2 & 11 RESPECTIVELY.
2. AUDIO AMPLIFIER
3. ELECTRO-DYNAMIC TYPE SPEAKER WITH 0.4 OHM VOICE COIL
4. GAGE-30 PLATINUM SUPPORT WIRES, EA. ABOUT 10 INCHES LONG
5. SPECIMEN (CYLINDER)
6. PICKUP, A GENERAL ELECTRIC VARIABLE RELUCTANCE PHONOGRAPH CARTRIDGE WITH SHIELDED CABLE TO 7
7. HEATHKIT PREAMPLIFIER - MODEL WA-PL.
8. HEATHKIT 20 WATT AMPLIFIER, MODEL A-5
9. CATHODE RAY OSCILLOSCOPE
10. VACUUM TUBE VOLTMETER
11. GENERAL RADIO AUDIO FREQUENCY METER TYPE 1141-A

FIG. 1 SONIC MODULUS OF ELASTICITY APPARATUS.

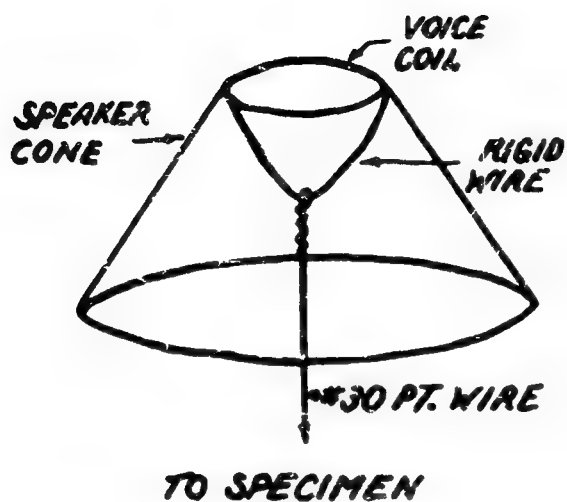


FIG. 2 DRIVING ASSEMBLY

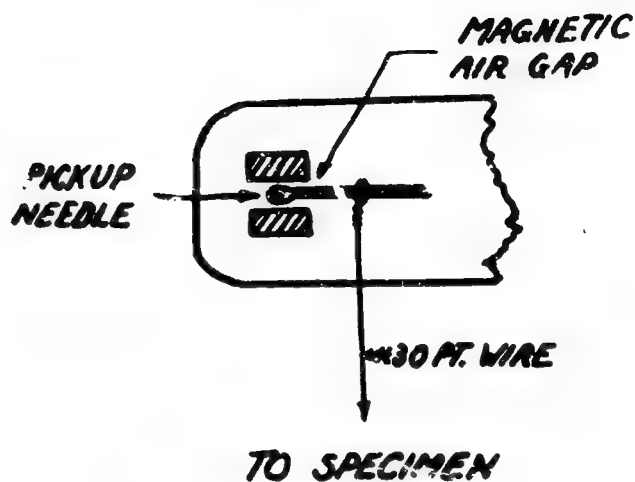


FIG. 3 PICKUP ASSEMBLY

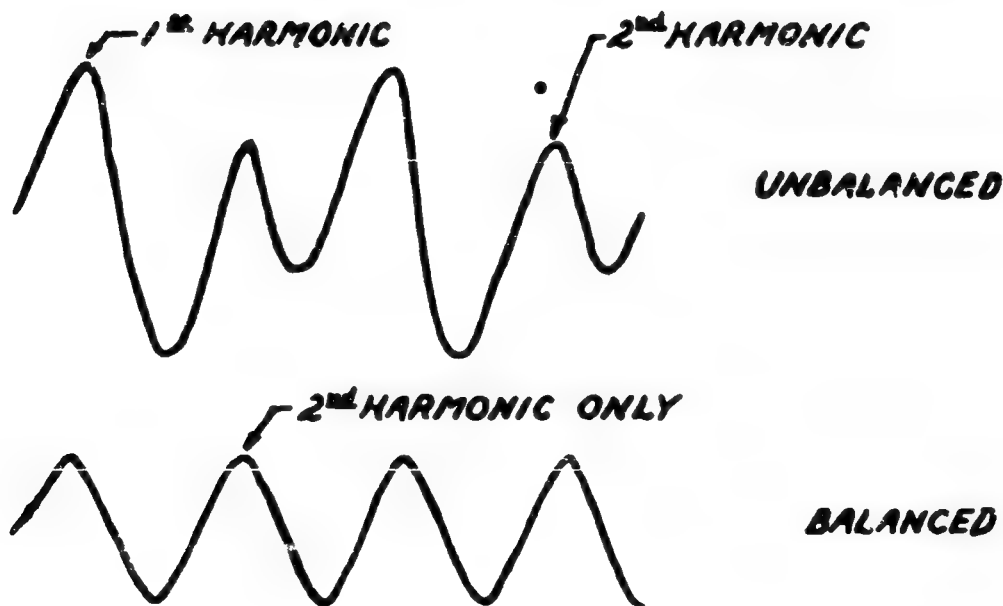


FIG. 4 OSCILLOSCOPE PATTERN FOR BALANCING FREQUENCY METER

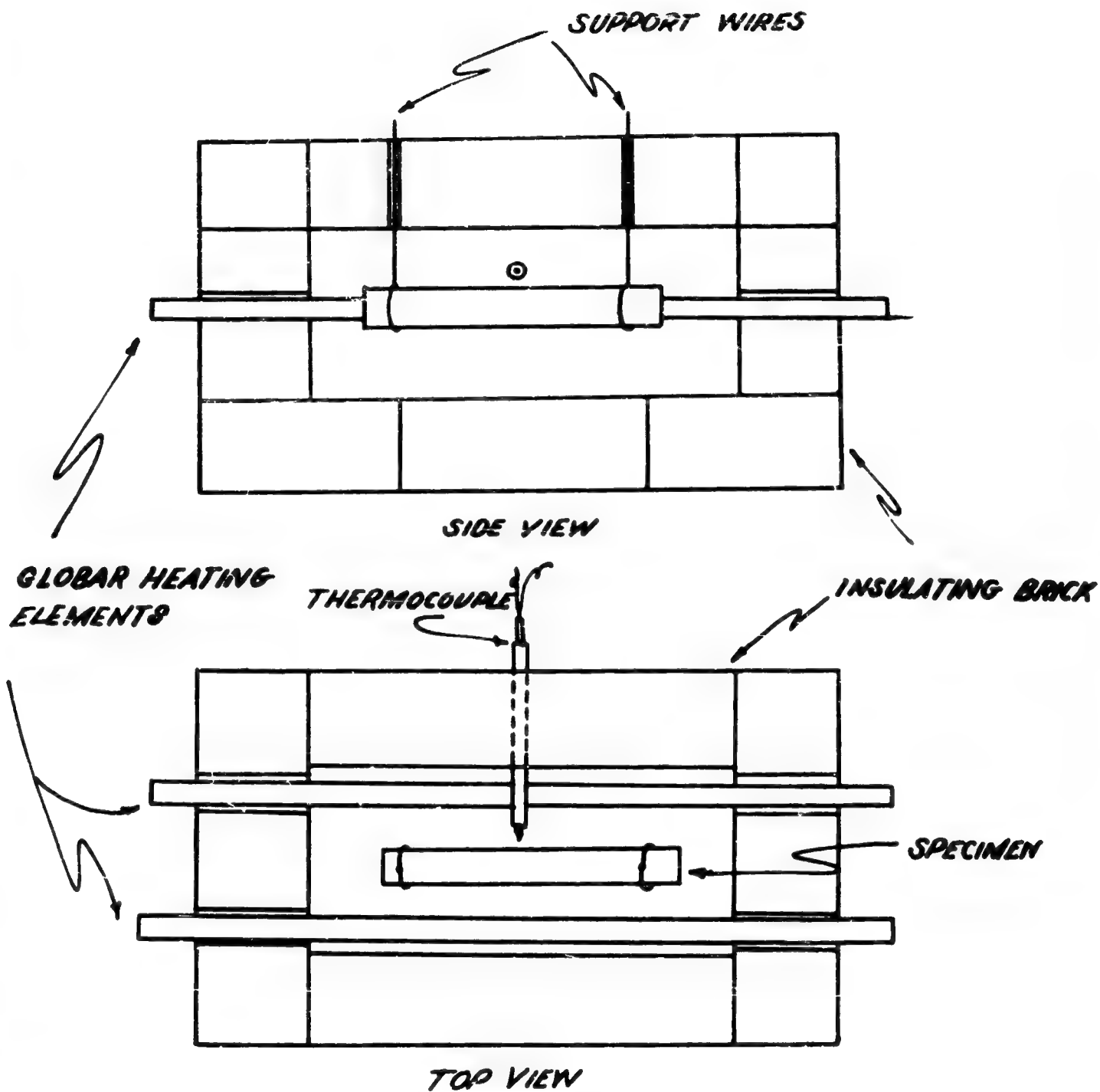


FIG. 5 FURNACE FOR SONIC E MEASUREMENT

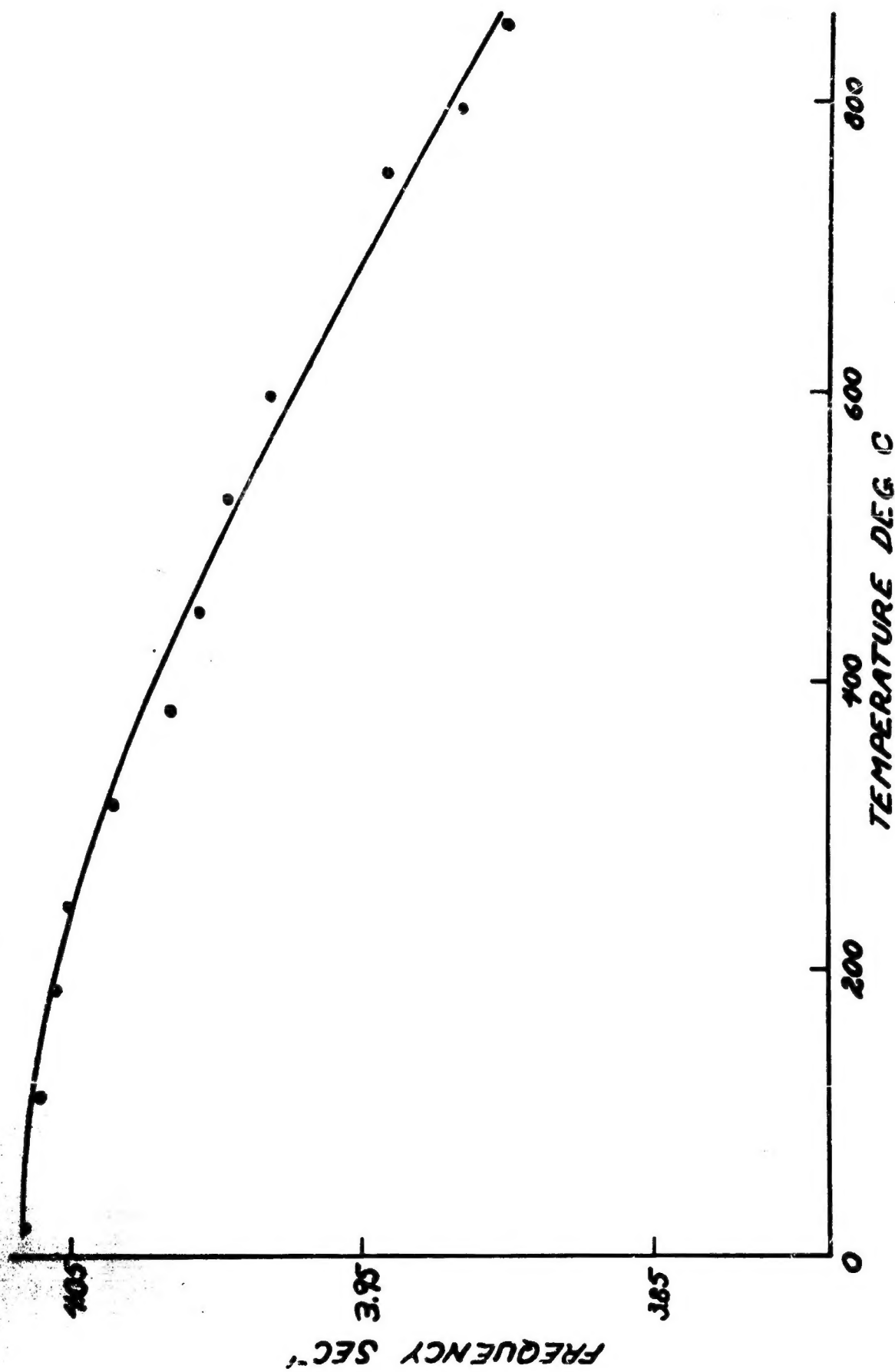


FIG. 6 RESONANT FREQUENCY VS TEMPERATURE

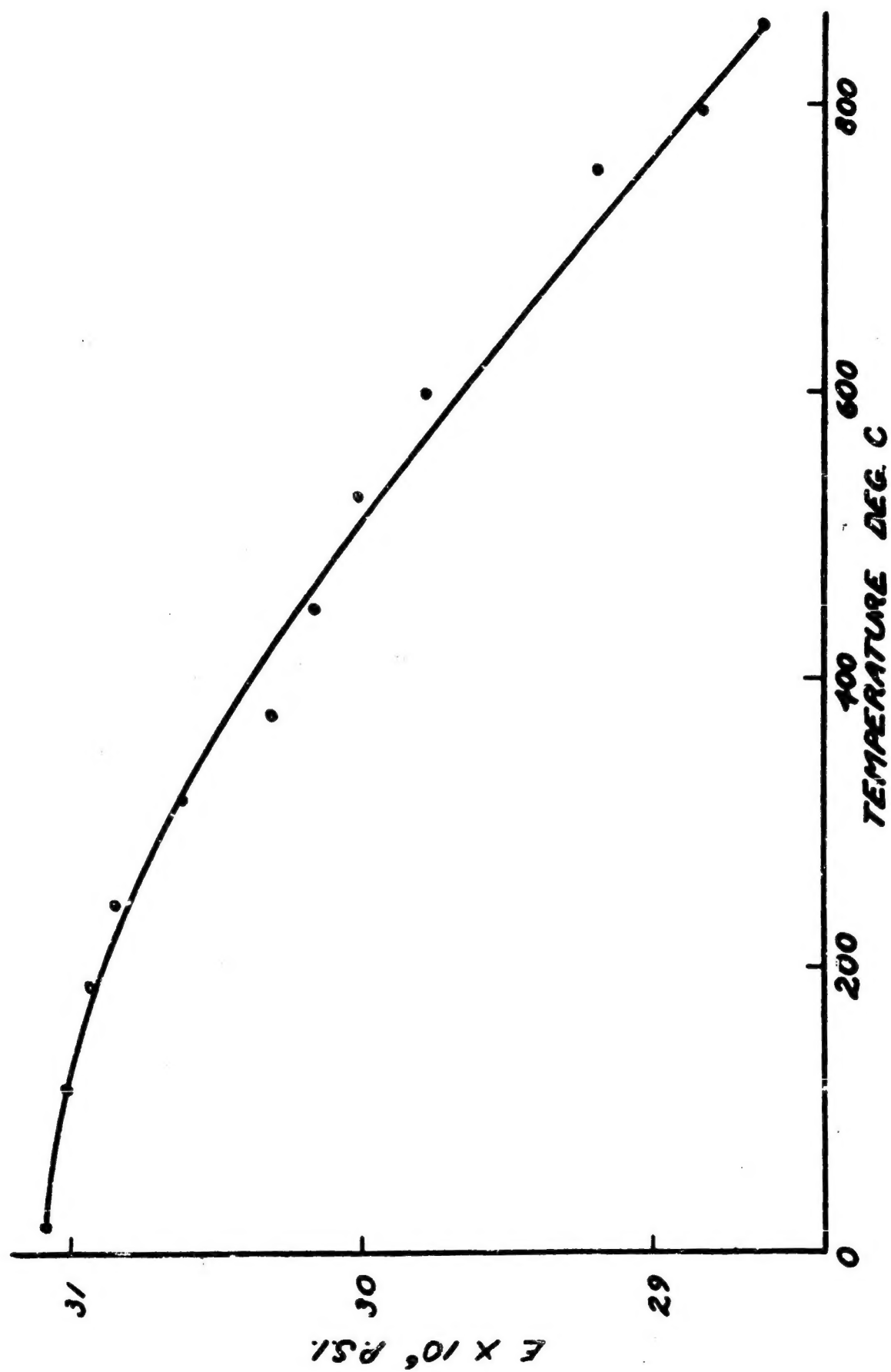


FIG. 7 YOUNG'S MODULUS VS. TEMPERATURE

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